Substitution and income effect for cobb-douglas

Given the utility function of a consumer: $U = 6 \cdot x^{1/3} \cdot y^{1/4}$, where p_x and p_y are the prices, and $M = \$1000$:

1. Find the Marshallian demands for both goods.
2. Find the indirect utility function.
3. Calculate the optimal bundle and utility when $p_x = 2$ and $p_y = 2$, and then when $p_x = 4$ and $p_y = 2$.
4. Calculate the total, substitution, and income effects according to Slutsky's equation.

Solutions

1.

$$L = 6x^{1/3}y^{1/4} + \lambda(1000 - p_x x - p_y y)$$
$$L'x = 2x^{-2/3}y^{1/4} - \lambda p_x = 0$$
$$L'y = \frac{3}{2}x^{1/3}y^{-1/3} - \lambda p_y = 0$$
$$L'\lambda = 1000 - p_x x - p_y y = 0$$

With the first two equations:

$$\frac{2x^{-2/3}y^{1/4}}{p_x} = \frac{3}{2p_y}x^{1/3}y^{-1/3}$$

$$\frac{2x^{-2/3}y^{1/4}}{p_x} = \frac{3}{2p_y}x^{1/3}y^{-1/3}$$

$$y = \frac{3xp_x}{4p_y}$$

Inserting it in the last equation:

$$1000 - p_x x - p_y \frac{3xp_x}{4p_y} = 0$$
$$1000 = \frac{7xp_x}{4}$$
$$x^m = \frac{1000}{p_x}\frac{4}{7}$$

Now inserting in y equation:

$$y = \frac{3p_x}{4p_y}\frac{1000}{p_x}\frac{4}{7}$$
$$y^m = \frac{1000}{p_y}\frac{3}{7}$$

2.

$$V = 6\left(\frac{1000}{p_x}\frac{4}{7}\right)^{1/3} \cdot \left(\frac{1000}{p_y}\frac{3}{7}\right)^{1/4}$$

3. When $p_x = 2$ and $p_y = 2$

$$x^m = 285.7020$$

$$y^m = 214.2980$$

$$V = 151.688$$

When $p_x = 4$ and $p_y = 2$

$$x^m = 142.85100$$

$$y^m = 214.2980$$

$$V = 119.9856$$

4.

$$\text{Total effect: } 142.8510 - 285.7020 = -142.851$$

And

$$\text{Total effect: } = \text{Substitution effect } + \text{Income effect}$$

Slutsky's substitution effect can be found by maintaining the purchasing power constant. This means using a new M' that allows the consumer to purchase the same bundle as before the change in prices.

$$M' = 4 \cdot 285.7020 + 2 \cdot 214.2980 = 1571.404$$

$$x(4, 2, M') = \frac{1571.404}{4}\frac{4}{7} = 224.4863$$

$$\text{Substitution effect: } 142.8510 - 224.4863 = -81.635$$

$$\text{Income effect: } -142.851 + 81.635 = -61.216$$

$$\text{Total effect: } = -81.635 + -61.216 = -142.851$$

If we calculate the substitution effect while maintaining the utility constant, the result is as follows:

$$119.9856 = 6\left(\frac{M'}{4}\frac{4}{7}\right)^{1/3} \cdot \left(\frac{M'}{2}\frac{3}{7}\right)^{1/4}$$

Solving for M'

$$M' = 1494.2743$$

$$x(4, 2, M') = \frac{1571.404}{4}\frac{4}{7} = 213.4678$$

$$\text{Substitution effect: } 142.8510 - 213.4678 = -70.6168$$

$$\text{Income effect: } -142.851 + 70.6168 = -72.2342$$

$$\text{Total effect: } -72.2342 - 70.6168 = -142.8502$$